THE EFFECTS OF DIFFERENT THREE INSTRUCTIONAL METHODS ON THE ABILITY TO COMMUNICATE MATHEMATICAL REASONING TO THE SMA STUDENTS

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ABSTRACT

Doing math is important, but understanding and communication what are they doing is more important. Because of that mathematical comminication very important to development to our student. The main purpose of the present study was to investigate the differential effects of Think-Talk-Write (TTW) Strategy and Cooperative Learning on the ability to communicate mathematical reasoning. Participants were 278 Senior High School (SMA)'s students who studied the "Square function graph" unit under three instructional methods namely: Cooperative Learning embedded within Think-Talk-Write (TTW) Strategy (COOP+TTW), Individualized Learning embedded within TTW Strategy (IND+ TTW), and Individualized Learning with no TTW Strategy (IND). Results showed that the Cooperative Learning embedded within TTW Strategy (COOP+TTW) group significantly outperformed the IND+TTW group, who in turn significantly outperformed the COOP and IND groups on various aspects of verbal explanations on graph interpretation test.

Key world: Doing Math, COOP+TTW, IND+TTW, and IND.

I. INTRODUCTION

Interest in communication is both more widespread and more central to mathematics education reform efforts than ever before. The NCTM (1989) reforms emphasize the importance of problem solving and communicating mathematical ideas, not simply isolated answers. Mathematical communication "requires attaining abilities to read, write, explain, discuss, justify, and clarify mathematical reasoning using different forms of representations" (Elliot & Kenney, 1996.p.ix.). Nevertheless, recognizing the centrality of communication as an issue for mathematics education is necessary but not sufficient to ensure a higher frequency of communication. Even when there is a high level of interest of commitment to communication as a feature of mathematics. Instruction, many teachers may struggle with the challenges arising from implementing these beliefs in classroom (Silver & Smith, 1996). There is an important need to investigate different instructional that can contribute to the attainment of mathematical communication in the classroom. Communication is the essence of the small-group experience. To foster the ability to communicate mathematical reasoning it is only natural to give students the opportunity to study in small groups where there interactions are enhanced. This is in contrast to "traditional instructions (which) places most students in a position of almost total dependence on the teachers. Student seem to learn by listening and watching the teacher do mathematics and than by trying to solve the problems on their own" (Frederiksen, 1994,p.536) rather than by being involving in mutual reasoning and resolution of cognitive conflicts that arise during the interactions. Research has suggested, however, that for positive outcomes to occur, small-group activities must be structured to maximize the chances that students will engage in questioning, elaborating, explanation, and other verbalizations in which they can express their ideas and through which the group members can give and receive feedback (Slavin, 1989).

These recommendations have led researchers King (1994); Mevarech & Kramarski, (1997) to suggest the structuring of group interaction through metecognitive training called in the IMPROVE method. Than Huinker & Laughlin, (1996) suggest the structuring of group interaction through TTW Strategy that enhances students understanding of the task, awareness and self-regulation of strategy application, and connections made between prior and new knowledge.

The method of Huinker & Laughlin, (1996), called "Talk Your Way Into Writing" emphasizes the importance of mathematical communication throughout the entire curriculum by changing classroom organization into small groups, learning and providing each student with the opportunity to do mathematics by involving him or her in mathematical communication via the use of TTW question namely to read, write, explain, discuss, justify, and clarify mathematical reasoning that focus on: (a) the nature of the problem (b) the construction of the relationships between previous and new knowledge solved in the past ' and (c) the use of strategies appropriate for solving the problem.

The purpose of the present study is to investigate the differential effect of TTW strategy (COOP+TTW) would facilitate the ability to communicate mathematical reasoning more than being exposed to individualized learning embedded within TTW strtategy (IND+TTW) which in turn would facilitate mathematical communication more than individualized (IND) setting with no TTW strategy.

II. METHOD

2.1. Participants:

Participants were 278 students (103 boys and 175 girl) who studied in ten grade classroom senior high school randomly selected, three classes in each treatment. The schools were an integrated school composed of students from different school level status in Pidie District.

2.2. Measures:

The ability to communicate mathematical reasoning was assessed by the graph interpretation test focusing on analyzing verbal explanation. To assess student ability to interpret graph and particularly square graphs, the test involved 10 short open ended items regarding basic knowledge about the square equation and square fungtion graph interpretation. The test involved items that required qualitative and quantitative graph interpretation skill. The short open ended items asked student to give final answer and to explain their reasoning in writing.

2.3. Scoring:

For each item, students received a score for either 10 (full correct answer) or 0 (incorrect answer), and total score ranging from 0 to 100. Kude Richardson reliability coefficient was 0.91.

2.4. Verbal explanations:

Each items on mathematical explanation was scored on three dimensions: correctness, fluency in providing different kinds of correct explanations and mathematical representations.

2.5. Correctness:

Explanations could be correct or incorrect, supported by different kinds of arguments, and formulated by formal or informal mathematical language. An explanation was considered as correct if the argument fit the conventions, even if it was not stated in a formal way. For example for linear fungtion graph, if a student argued that the chance-rate of line A is greater than the change-rate of line B because “line A is steeper than line B”, that argument was considered even though it was not phrased with formal mathematical concepts such as slope. Scoring: For each item, student received a score for either 10 (correct explanation) or 0 (incorrect explanation), and total score ranging from 0 to 100.

2.6. Fluency in providing different kinds of correct explanations:

Students could use one or more arguments to explain their reasoning. Scoring: The number of correct explanations student provided for each item.

III. MATHEMATICAL REPRESENTATIONS

Students mathematical explanations were classified into four categories: (a) verbal arguments based on visual analysis of the graph, For example for linear fungtion graph (line is A steeper, line A is more diagonal); (b) verbal arguments based on formal concepts (the change-rate of line A is bigger because its slope is steeper than of line B); (c) numeric/algebraic arguments (the change-rate of line A is three time more than the change rate of line B); and (d) arguments based on drawings that students added to the graph(adding one-unit steps to the graph and calculating the change rate by using the steps). Two judges who are expert in mathematics education analyzed students’ explanations. Inter judge reliability coefficient was 0.88.

IV. TREATMENT

All classroom studied the square fungtion graph unit three times a week for four weeks. In particular, in all classroom students studied; (a) the concept of square equation and square fungtion, solution the square equation; (b) quantitative and qualitative method of graph interpretation; and (c) transformation of algebraic expressions of the from $y = ax^2 + bx + c$ into graphic representations.

The TTW instruction used in the present study is based on the techniques suggested by Huinker & Laughlin, (1996), called "Talk Your Way Into Writing". The TTW instruction utilizes a series of self-addressed think-talk-write questions, connections questions and reflection questions. In addressing comprehension questions, student had to read the problem, describe the concepts in their own words, and try to understand what the concepts meant. The strategic questions are designed to prompt students to consider which strategies are appropriate for solving the given problem and for what reasons.

Connections questions prompt students to focus on similarities and differences between the graph at hand and graphs they had already interpreted or to compare different intervals on the same graph. In doing so, students gradually learn to construct a network of information or a schema for understanding. Reflection questions prompt students to focus on the solution process and to ask themselves" what am I doing here?", "does it make sense?", "what if?" the thik-talk-write questions were printed in students booklets, teacher guide and the held index cards the students used in problem solving. Students used the think-talk-write question orally in their small group/individualized activities, and writing when they use their booklets.

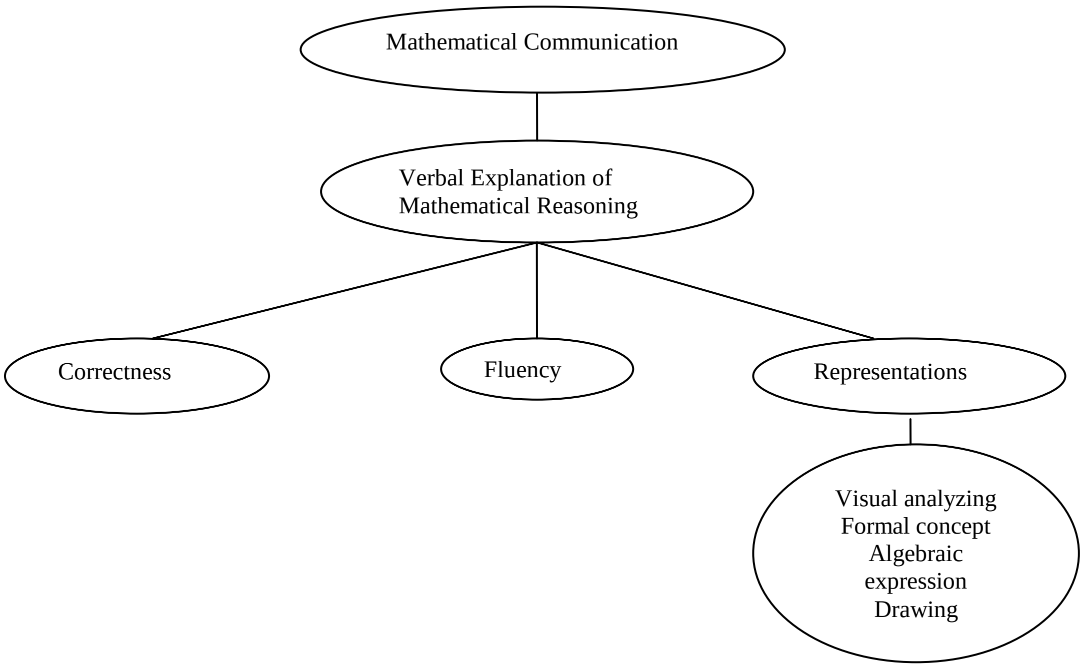

Figure 1: Summarizes The Relationship Addressed Under Mathematical Communication in This Study.

V. LERNING INSTRUCTIONS

COOP+TTW condition: students in this condition studied in small heterogeneous groups composed of four students: one high achiever, one low achiever, and two middle achiever, using the metacognitive question describe above. IND+TTW condition: In this condition, the TTW instruction was exactly the same as is the above condition, expect that the metacognitive instruction was implemented in individualized rather than cooperative settings.

COOP condition: Under thiis condition, student studied in small heterogeneous group as in the COOP+TTW condition, but they were not exposed to the Thin-Talk-Write Strategy. IND condition: Under this condition students learned individually with no TTW Strategy. This group served as a control group.

VI. RESULT

ANCOVA analysis was performed on graph interpretation achievements and on the various aspects of verbal explanations controlling for pretreatment differences.

Graph interpretation: Table 1 indicated that although no significant differences were found between treatment group prior to the beginning of the study, significant differences were found at the end of study. Post-hoc analysis of the adjusted mean scores based on pair wise technique indicated that the COOP+TTW group significantly outperformed the COOP and IND groups, but no significant differences were found between the two groups who were not exposed to the TTW strategy.

Table 1: Mean Scores, Adjusted Mean Scores, and Standard Deviations on Graph Interpretation Test

		COOP+TTW N= 90	IND+TTW N= 90	IND N= 90	F (4,380)
Pretest	M:	35,5	34,4	36,0	1,86
	S :	6,4	6,4	6,2	
Posttest	M:	64,4	60,9	59,8	18,44**
Adjusted	M:	64,0	61,4	59,1	
	S :	7,2	6,9	6,6	

**p<0,005

Verbal explanations: Table 2 indicated that although no significant differences were found between groups prior were beginning of the study on both correctness and fluency, significant differences were found at the end of study. Yet, post-hoc analysis based on the pair wise technique indicated differences pattern of performance on both measures. On correctness, the COOP+TTW groups out performed all other groups, but no significant differences were found between the IND+TTW, and IND groups. On fluency, the COOP+TTW groups out performed the IND+TTW group who in turn significantly outperformed and IND groups, but no significant differences were found on that measure between the two groups who were not exposed to TTW strategy.

Table 2: Mean Scores, Adjusted Mean Scores, and Standard Deviations on Verbal Explanations

		COOP+TTW N= 90	IND+TTW N= 90	IND 9= 30	F (4,380)
Correct explanation					
Pretest	M:	29	27	31	0,32
	S :	2,5	2,3	2,6	
Posttest	M:	65	44	41	26,43**
Adjusted	M:	65	45	40	
	S:	3,1	3,1	2,6	
Fluency in providing different kinds of correct explanations					
Pretest	M:	32	28	37	2,26
	S:	2,4	2,2	2,6	
Posttest	M:	89	65	46	22,55**
Adjusted	M:	89	67	42	
	S:	5,0	4,4	2,9	

**p<0,005

Mathematical representations: Table 3 indicated that most students relied on numerical and algebraic representation and justifying their reasoning. Interestingly, the individualized group (with or without TTW strategy) did so even more frequently then the cooperative groups (with TTW strategy). In addition to using numerical and algebraic representations quite often students used verbal- formal representations. The frequency to using verbal formal representations, however, was significantly large under the COOP+TTW condition than under all other conditions. These differences were statistically significant (Chi square = 27,0, p<0,0005). Further analysis showed the under all conditions, student used the visual and graphic representations quite infrequently (less than 5% of the student).

Table 3: Frequencies (percent in parentheses) of student Who Used Mathematical Representations in Justifying Their Correct Mathematical Reasoning

	COOP+TTW N= 90	IND+TTW N= 90	IND N= 90
Visual explanations			
Pretest	60(1,6)	30(0,8)	50(1,3)
Posttest	70(1,8)	70(1,3)	60(14)
Formal explanations			
Pretest	19(4,9)	16(14,7)	18(4,7)
Posttest	31(8,1)	11(2,9)	19(4,9)
Numeric/algebraic explanations			
Pretest	50(13,0)	46(12,00)	40(10,4)
Posttest	54(14,1)	61(15,9)	60(15,6)
Drawing			
Pretest	10 (0,4)	10 (0,2)	10(0,3)
Posttest	60(1,6)	50(1,3)	30(1,1)
Total Mean			
Pretest	34,8(19,5)	25,5(16,9)	29,5(16,7)
Posttest	53,8(25,5)	48 (21,8)	42,3(20,6)

VII. CONCLUSIONS

It was found that cooperative learning embedded with metacognitive training is effective in developing the ability to communicate mathematical reasoning in the classroom on three dimensions of verbal explanations: correctness, fluency and representations. The result indicate that verbal explanations improve understanding on graph interpretation. These findings support earlier conclusions. Cohen (1996) indicated that features of discourse are new behaviors that student can learn through practice and reinforcement. “Giving reasons for ideas”, for example, can become a norm of behavior than enhance mathematical thinking and communication.

Mavarech & Kramarski (1997) state the being presented with explanations related to why and how a certain solution to a problem has been reached, the student is given the opportunity to elaborate upon the information inherent in the explanations, and thus learn from them. More theoretical conclusions and practical implications will be discussed on the presentation. In addition their will be presented more details on the Think-Talk-Write Strategy and examples of students verbal explanations regarding each instructional method.

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